Developing a Predictive Failure Model for Polyurethane Treaded Wheels Based on Loads and Speeds

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Introduction

Problem
- Current load ratings for polyurethane treaded wheels are vague
  - Intermittent use
  - Walking speeds
- Simulation testing necessary for any prolonged dynamic application

Solution
- Need a theoretical model to predict wheel/tire failure
Introduction

• Polyurethane generates heat when cyclically deformed
  - Heat generated due to hysteresis of the material

• Excessive loads or speeds cause urethane tires to generate heat faster than it can be dissipated
  - Leads to break down of bond
  - Eventual tire failure

Introduction

• Benefits of a predictive failure model
  - Eliminates need for costly application simulation testing
  - Allows the correct wheel/tire combination to be selected for various applications
  - Means for comparing various urethane tires without need for numerous tests
Developing a Predictive Failure Model

• Energy Balance

  $E_{\text{Generated}} = E_{\text{Absorbed}} + E_{\text{Out}} \ (J)$
  
  - Energy generated from tire deformation
  - Energy absorbed to raise internal energy of wheel core
  - Energy leaves system through convective heat loss

Developing a Predictive Failure Model

• Energy Generated
  
  - Due to cyclic deformation of urethane tread
    - Based on tread deflection formula
  - Depends on urethane, wheel load, and speed

  • Deflection of tire can be modeled by:

  \[ U = \left( \frac{0.75 \ L_s}{Ew \ (8 \ R)^{1/2}} \right)^2 \ (m) \]
Developing a Predictive Failure Model

- Energy Generated
  - Assume the energy created by the deformation of tire is similar to potential energy of a compressed spring
  \[ E_{\text{GenSpring}} = \frac{1}{2} k_{\text{spring}} U^2 \] (J)
  - Assume a portion of that elastic spring energy is converted into thermal energy (for cyclic loading, must also include angular frequency of rotation and duration of use)
  \[ E_{\text{Generated}} = k_{\text{HG}} \left( \frac{v}{R} \right) t \left( E_{\text{GenSpring}} \right) \] (J)

- Then solve:
  \[ E_{\text{Generated}} = k_{\text{HG}} \left( \frac{s}{w} \right)^{4/3} \frac{v}{5} \left( \frac{L^3}{R^3} \right) \] (J)
  - All constants are lumped into \( k_{\text{HG}} \), the heat generation coefficient
Developing a Predictive Failure Model

- Energy Out
  - Convective heat loss for each face of the wheel can be modeled as:

\[ E_{Out} = hA\Delta T_{Out} \quad \text{(J)} \]
• Energy Absorbed
  - Energy absorbed (stored) in the wheel core is modeled as:
  \[ E_{\text{Absorbed}} = MC_p \Delta T \quad (J) \]
  - \( C_p \) for cast iron: 460 J/kg°C
  - \( M \) (mass in kg) measured for each core size

• Energy Balance
  - To evaluate the energy flux of the system, take the derivative with respect to time
  \[ k \left( \frac{s}{w} \right)^{4/3} \frac{vtL}{R^3} = 2hAt\Delta T_{\text{out}} + MC_p \Delta T \quad (J) \]
  - To evaluate the derivative with respect to time
  \[ \frac{d}{dt} \left( k \left( \frac{s}{w} \right)^{4/3} \frac{vtL}{R^3} \right) = 2hAt\Delta T_{\text{out}} + MC_p \Delta T \quad (W) \]
Developing a Predictive Failure Model

• Energy flux balance:

\[
k_{HG} \left( \frac{R^3}{W} \right)^{\frac{4}{5}} vL^3 \frac{dT_{Wheel}}{dt} = MC_p \frac{dT_{Wheel}}{dt} + 2hA(T_{Wheel} - T_{Ambient}) \ (W)
\]

• Boundary conditions: \( T_{Wheel(t=0)} = T_{initial} \) and \( T_{Wheel(t=8)} = T_8 \ (°C) \)

• Solving the ODE yields:

\[
T_{Wheel(t)} = \left( T_{Initial} - T_\infty \right) e^{MC_p \frac{-2hA t}{}} + T_\infty \ (°C)
\]

Developing a Predictive Failure Model

• Solving the ODE as time \( \to 8 \)

\[
\frac{1}{2hA} \left( k_{HG} \left( \frac{R^3}{W} \right)^{\frac{4}{5}} vL^3 \frac{4}{5} \right) + T_{Ambient} = T_\infty \ (°C)
\]

• Now with a formula that predicts final steady state wheel temperature, only the failure temperature of the urethane is needed
Testing

- To validate the formula, a series of tests were run on CCI’s Dynamic Wheel Endurance Tester (DWET).

- All tests were run with CCI’s standard 85 Shore A Polyester MDI material.

Testing

- All wheels had a thin tread tire (3/8 in or 0.01 m).

- Wheel diameters tested were 6, 8, and 10 inches (0.15, 0.20, and 0.25 m respectively).

- Wheels were tested in widths of 2 and 3 inches (0.05 and 0.08 m respectively).

- Total of 30 tests were run.
Testing

• Determining failure temperature of CCI 85 A MDI Polyester Urethane

6x3 85 A MDI Wheel Running at 6,672 N and 0.89 m/s (1500 lbs and 2 mph)

• Wheels that failed all exhibited the same spike in torque
  • The spike occurred when wheels were near 60°C
  • Failure temperature was found to be 60±5°C for CCI 85 A MDI Urethane

Solving for the two unknowns:

• To solve for \( hA \):
  \[
  hA = -\frac{MC_p}{2t} \ln \left( \frac{T_W(t) - T_{\infty}}{T_{\text{initial}} - T_{\infty}} \right) \quad \text{(W/°C)}
  \]

• To solve for \( k_{HG} \):
  \[
  k_{HG} = \frac{2hA(T_{\infty} - T_{\text{Ambient}})}{\frac{4}{3} \frac{V}{R^\frac{2}{3}} \left( \frac{S}{W} \right) ^\frac{2}{3}} \quad \text{(J-m²/°C)}
  \]
Testing

• When wheels did not fail, they exhibited temperature profiles as displayed in the following graph:

8x2 85 A MDI Wheel Running at 2224 N and 2.68 m/s (500 lbs and 6 mph)

• From these graphs, $T_{\text{initial}}$, $T_8$, and $T_W(20\min)$ were recorded

Results

• Validation of lumped capacitance assumption
  • Biot number must be $\leq 0.10$

\[
Bi = \frac{R_{\text{Conduction}}}{R_{\text{Convection}}}
\]

• Biot number is equal to $0.10\pm0.01$ at wheel speeds of 2.68 m/s (6 mph)
• Lumped capacitance model is valid for our experiments
Results

• Development of \( hA \) equation
  • The \( h \) varies as wheel speed changes
  • The \( A \) varies as wheel surface area changes
  • Need to predict \( hA \) based on speed and size of wheel
  • Standard convective heat loss formula for a spinning vertical disk:

\[
    hA = \pi k_f R \left( 0.3 \left( \frac{2}{3} \sqrt{R} \sqrt{v} \right) \right) \quad (\text{W/}^\circ\text{C})
\]

• Initial assumption was natural convection heat loss was negligible

• Comparison of theoretical \( hA \) to experimental \( hA \) values
  • Theoretical matches well with experimental when a 0.54 W/°C correction factor is added in

\[
    hA_{\text{corr}} = (0.048v + 0.599) + 0.078 \quad \text{W/}^\circ\text{C}
\]

\[
    hA_{\text{exp}} = 0.057v + 0.583
\]

\[
    hA_{\text{theor}} = (0.048v + 0.068) + 0.060 \quad \text{W/}^\circ\text{C}
\]

• Measured natural convection losses on a still wheel at 0.42 W/°C
• Initial assumption of negligible natural convection loss was wrong
Results

Finding $k_{HG}$ – the heat generation coefficient

- The heat generation coefficient is primarily material dependant
  - It will vary if the chemical bond between urethane and core material is poor or not consistent

- For CCI 85 A MDI Polyester urethane:
  \[ k_{HG} = 5.5 \times 10^{-5} \pm 1.5 \times 10^{-5} J\cdot m^{2/3}/N^{4/3} \]
  - Will vary with every urethane
  - The higher the $k_{HG}$ value, the more heat the urethane generates

Results

Verification of Predictive Failure Equation (PFE)

- Will verify equation by comparing it to experimental results

Actual vs. theoretical temperature profile of an 8x2 85 A Polyester MDI treaded wheel running at 2.7 m/s and 2,225 N (6 mph and 500 lbs).
Results

Actual vs. theoretical temperature profile of an 10x3 85 A Polyester MDI treaded wheel running at 2.8 m/s and 5,562 N (6 mph and 1250 lbs).

Results

Actual vs. theoretical temperature profile of an 8x2 85 A Polyester MDI treaded wheel running at 2.24 m/s and 3,560 N (5 mph and 800 lbs).
Conclusions

- PFE predicts steady state temperature of urethane treaded wheels accurately within ±5% of experimental results

\[
\frac{1}{2hA}(k_{HG} \left( \frac{s}{w} \right)^{\frac{4}{3}} \frac{vL_3}{R^3} ) + T_{Ambient} = T_\infty \pm 5\% \ (°C)
\]

- Only a small number (1-2) of tests needs to be run to predict product ratings for a specific urethane tread.

- Some drawbacks
  - Does not account for heat loss from air passing over wheel in translational motion
  - Does not account for floor roughness or frictional heat due to “scrubbing”
Uses

- PFE can be used to choose the correct tire and wheel combination for specific applications.
- The heat generation coefficient \(k_{HG}\) and failure temperature can be used to compare various urethanes.
- The \(k_{HG}\) can also be used as a quality control tool.
- Cannot be used for:
  - Thick tread urethane tires
    - Urethane is an insulator so excess heat is stored in the tire
  - Polymer wheel cores
    - Polymers would insulate the tire causing more heat to build up in the tire
  - When tire deflection is greater than 10% of thickness
    - Urethane is then overstressed and heat generation grows exponentially

Thank You

Questions?

\[
\frac{1}{2h} (k_{HG} \left( \frac{s}{W} \right)^{\frac{4}{3}} \left( \frac{vL}{R^3} \right)^{\frac{4}{5}} + T_{Ambient}) = T_a \pm 5\% \text{ (°C)}
\]

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